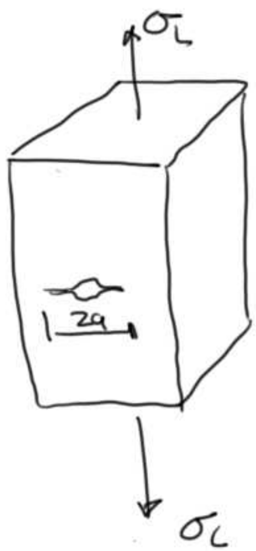
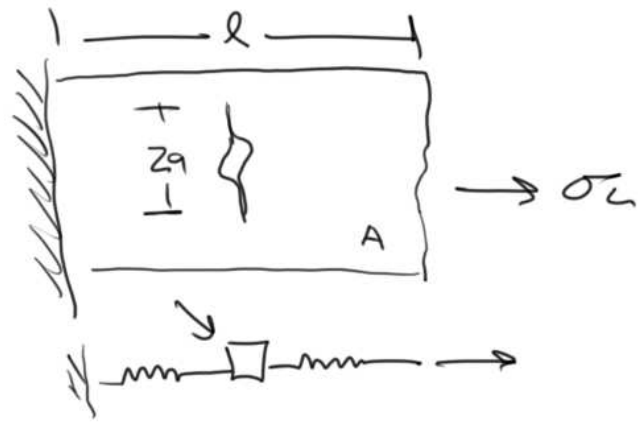
HF Overview

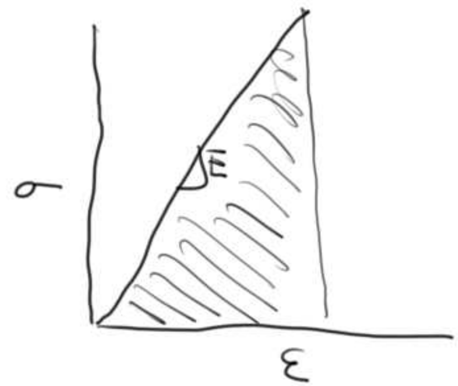
- Fracture Mechanics
- Analytic Models
- Computer Models

Griffith - 1921



$$c \propto \sigma_c \sqrt{a}$$





$$U_E + U_S - W_L = 0$$

$$\frac{\partial U_S}{\partial a} + \frac{\partial}{\partial a}\left(U_E - W_L\right) = 0 \quad \Rightarrow \quad \boxed{\frac{\partial U_S}{\partial a} = \frac{\partial U_E}{\partial a}}$$

$$W_L = F \cdot \text{distance}$$

$$= F \cdot \Delta l$$

$$= (\sigma_L A) \cdot (\varepsilon l)$$

$$\varepsilon = \frac{\Delta l}{l}$$

$$U_E = \frac{1}{2} \sigma_L \varepsilon A l$$

$$\frac{W_L}{U_E} = \frac{\sigma_L \varepsilon A l}{\frac{1}{2} \sigma_L \varepsilon A l} = 2 \quad \Rightarrow \quad W_L = 2U_E$$

$$U_s = 2 \cdot 2a \cdot \gamma \cdot \text{width} \quad \Rightarrow \quad \frac{U_s}{\text{width}} = 4a\gamma$$

↑ specific surface energy

$$\frac{U_E}{\text{width}} = \pi a^2 \sigma_c^2 \left[\frac{1-\nu^2}{E}\right] \quad \text{(Inglis, 1923)}$$

Plane strain $\quad \varepsilon_z \approx 0 = \frac{\Delta L}{L}$

$$\frac{\partial}{\partial a}\left(\frac{U_s}{w}\right) = \frac{\partial}{\partial a}\left(\frac{U_E}{w}\right)$$

$$\frac{\partial}{\partial a}(4a\gamma) = \frac{\partial}{\partial a}\left(\pi a^2 \sigma_c^2 \left[\frac{1-\nu^2}{E}\right]\right)$$

$$G = 2\gamma = \frac{\pi a \sigma_c^2 (1-\nu^2)}{E}$$

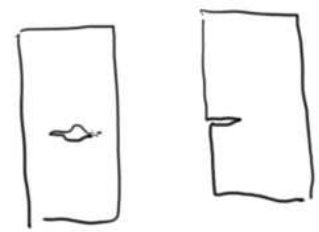
↳ strain energy release rate

$$\sigma_c = \sigma_c \quad \Rightarrow \quad G_c = \frac{\pi a \sigma_c^2 (1-\nu^2)}{E}$$

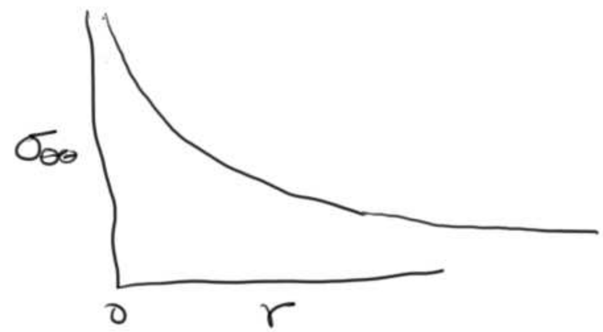
$$\sigma_c \sqrt{\pi a} = \left[\frac{G E}{(1-\nu^2)}\right]^{1/2}$$

$$K_I = \left[\frac{G E}{(1-\nu^2)}\right]^{1/2}$$ (Irwin, 1957) LEFM

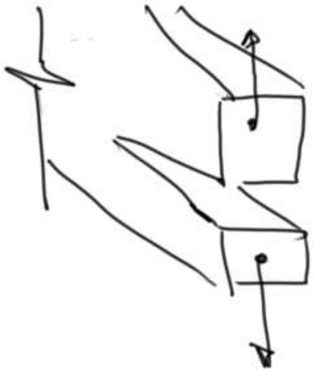
K_I = stress intensity factor



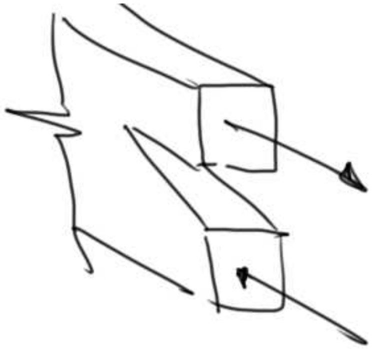
$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$



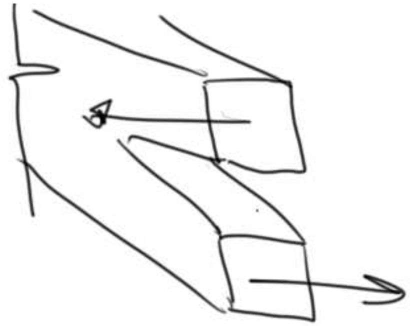
$$\sigma = E \varepsilon = E \frac{\partial u}{\partial x}$$



Mode I



Mode II



Mode III

$$G_c = \frac{(1-\nu^2)}{E} K_{Ic}^2 + \frac{(1-\nu^2)}{E} K_{IIc}^2 + \frac{(1+\nu)}{E} K_{IIIc}^2$$

J-integral

ASTM E399

$$2.5a \ll \frac{K_{Ic}^2}{\sigma_y^2}$$

HF

$$K_I = (\underbrace{p_f - \sigma_3}_{p_{net}})\sqrt{\pi L}$$

during propagation $p_{net} = \frac{K_{Ic}}{\sqrt{\pi L}}$