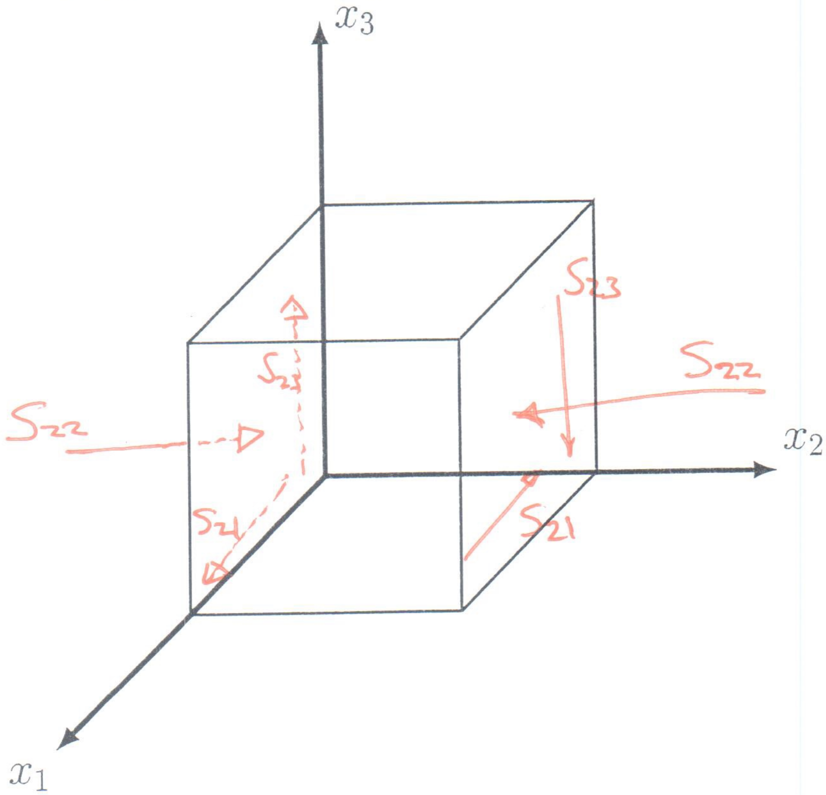
Problem 1

(**5 points**) Consider the following figure:



On the figure, correctly indicate (with hand-drawn arrows) the stress components S_{22}, S_{21}, and S_{23} on both sides of the cube where they act. **Use a "compression positive" sign convention.**

Problem 2

For the following matrix $\mathbf{A}$,

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$$

perform the following calculations **by hand**:

(i) (**5 points**) Compute the determinate of $\mathbf{A}$.

(ii) (**5 points**) Compute the eigenvalues of $\mathbf{A}$.

(iii) (**5 points**) Compute the eigenvectors of $\mathbf{A}$, put them in unit-vector form.

(i) $(1)(5) - (3)(4) = -7$ ///

(ii) $\det(A - \lambda I) = 0 \Rightarrow \det\left(\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$

$$\det\left(\begin{bmatrix} 1-\lambda & 3 \\ 4 & 5-\lambda \end{bmatrix}\right) = 0 \Rightarrow (1-\lambda)(5-\lambda) - (3)(4) = 0$$

$$\lambda^2 - 6\lambda - 7 = 0$$

$$(\lambda - 7)(\lambda + 1) = 0$$

$$\lambda_{1,2} = \{7, -1\} \quad ///$$

(iii) For $\lambda_1 = 7$

$$A - \lambda_1 I = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix} \xrightarrow{\frac{1}{4}R_2} \begin{bmatrix} -6 & 3 \\ 1 & -\frac{1}{2} \end{bmatrix}$$

$$\xrightarrow{\frac{1}{6}R_1} \begin{bmatrix} -1 & \frac{1}{2} \\ 1 & -\frac{1}{2} \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} -1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix}$$

$-v_1 + \frac{1}{2}v_2 = 0$

$v_2 =$ free, choose $2 \therefore v_1 = 1$

$$\vec{v}_{\lambda_1} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

///

This page provided for additional calculations

For $\lambda_2 = -1$

$$A - \lambda_2 I = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \xrightarrow[\frac{1}{4}R_2]{\frac{1}{2}R_1} \begin{bmatrix} 1 & \frac{3}{2} \\ 1 & \frac{3}{2} \end{bmatrix}$$

$$\xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 0 \end{bmatrix}$$

$v_1 + \frac{3}{2} v_2 = 0$

$v_2 = $ free, choose 2 $\therefore$ $v_1 = -3$

$$\vec{v}_{\lambda_2} = \begin{Bmatrix} -3 \\ 2 \end{Bmatrix} = \begin{Bmatrix} -3 \\ 2 \end{Bmatrix} \frac{1}{\sqrt{13}} \quad ///$$

Problem 3

(**20 points**) Given the following principle stresses under *strike-slip* faulting (using Anderson classification). What is the geographical stress tensor, $\mathbf{S}_G$, if S_{Hmax} is oriented exactly south?

$$S_1 = 60 \text{ MPa}, \quad S_2 = 50 \text{ MPa}, \quad S_3 = 45 \text{ MPa}$$

$\alpha = 180^\circ$

$\beta = 0^\circ$

$\gamma = 90^\circ$

From computer code:

$$S_G = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

This page provided for additional calculations

Problem 4

Given the geographical stress,

$$\mathbf{S}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \text{ MPa}$$

(i) (**10 points**) For a fault strike oriented northeast-southwest and a dip 65° from southeast, determine the normal and shear stresses acting on the fault plane.

(ii) (**5 points**) If the fault where to slip, what type of fault slip would it be?

(iii) (**10 points**) Faults typically slip when the ratio of shear to normal effective stress exceeds 0.6, i.e. $\tau/\sigma_n^{eff} > 0.6$. Using this criterion, calculate the critical injection pressure in petroleum engineering operations that one should not exceed to prevent fault slippage. You can assume that the fault is very near the injector and steady state conditions, such that the pore pressure is equal to the injection pressure.

(i) From computer code & strike $\approx 45^\circ$, dip $= 65^\circ$

$S_n = 56.4$ MPa

$\left.\begin{aligned} \tau_s &= 0 \\ \tau_d &= 7.6 \text{ MPa} \end{aligned}\right\} \quad \tau = 7.6$ MPa

(ii) Normal fault

(iii) $\dfrac{\tau}{\sigma_n^{eff}} > 0.6 \Rightarrow \dfrac{\tau}{(S_n - P_c)} = \mu$

$$P_c = \frac{\mu S_n - \tau}{\mu} = \frac{0.6(56.4) - 7.6}{0.6} \approx 43.6 \text{ MPa}$$