# Problem 1

Consider the following equation which describes the transverse deflection associated with a simply supported beam subject to a uniform transverse load $q(x) = q_0$,

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[EI\frac{\mathrm{d}^2 w(x)}{\mathrm{d}x^2}\right] = q_0 \quad \text{for} \quad 0 < x < L,$$

subject to boundary conditions,

$$w(0) = EI\frac{\mathrm{d}^2 w(0)}{\mathrm{d}x^2} = 0, \quad \text{and} \quad w(L) = EI\frac{\mathrm{d}^2 w(L)}{\mathrm{d}x^2} = 0.$$

### (a) 10 points

Develop and clearly indicate the weak form of this differential equation.

### (b) 5 points

Using the following approximation for $u$,

$$u \approx u_h = c_1 x\left(x - L\right),$$

use the Ritz method to determine the coefficient $c_1$.

### (c) 5 points

Using the following approximation for $u$,

$$u \approx u_h = c_1 \sin\left(\frac{\pi x}{L}\right)$$

use the Ritz method to determine the coefficient $c_1$.

### (d) 5 points

Compare your answers from (b) and (c) with the analytic solution at $x = L/2$. Which is more accurate? Why? (Hint: It has something to do with the boundary conditions.)

## Problem 2

We have seen that the following one-dimensional boundary value problem describes the physics of many interesting problems in engineering

$$-\frac{d}{dx}\left[a\frac{du}{dx}\right] + cu - f = 0 \quad \text{for} \quad 0 < x < L,$$

subject to the boundary conditions

$$u(0) = u_0 \quad \text{and} \quad \left[a\frac{du}{dx}\right]_{x=L} = Q_0,$$

where the $x$ is the independent variable, $u = u(x)$ is the dependent variable, and $a = a(x), c = c(x), f = f(x), u_0$, and $Q_0$ are the *data* of the problem.

### (a) 30 points

Write a general one-dimensional finite element code to solve this problem. General means that the user will input the *node* locations and specify the functions $a(x)$, $c(x)$, and $f(x)$ as well as the boundary conditions $u_0$ and $Q_0$. The output of the code should be the value of $u$ at the user specified nodes. You can restrict the code to using only linear interpolated elements.

You can verify your code with following data and results:

1. For $a(x) = 1 - x/2$, $c(x) = 0$, $f(x) = 0$, $u_0 = 0$, and $Q_0 = 1$,

| $x$ | $u(x)$ |
|---|---|
| 0.00 | 0.00000 |
| 0.25 | 0.26666 |
| 0.50 | 0.57435 |
| 0.75 | 0.93799 |
| 1.00 | 1.38244 |

2. For $a(x) = 1 - x/2$, $c(x) = x$, $f(x) = x$, $u_0 = 0$, and $Q_0 = 1$,

| $x$ | $u(x)$ |
|---|---|
| 0.00 | 0.00000 |
| 0.40 | 0.46695 |
| 0.60 | 0.73242 |

| $x$ | $u(x)$ |
|---|---|
| 0.65 | 0.80319 |
| 0.70 | 0.87615 |
| 1.00 | 1.38355 |

3. For $a(x) = 1$, $c(x) = 0$, $f(x) = x^2$, $u_0 = 0$, and $Q_0 = 2$,

| $x$ | $u(x)$ |
|---|---|
| 1 | 0.000 |
| 2 | 22.08 |
| 3 | 40.00 |
| 4 | 48.75 |

## (b) 20 points

A one-dimensional heterogenuous porous medium of length $L = 1$ m, has a steady state pressure distribution as shown in the following table.

| $x$ (m) | $p(x)$ (kPa) |
|---|---|
| 0.00 | 100.0 |
| 0.05 | 100.005 |
| 0.10 | 100.010 |
| 0.15 | 100.014 |
| 0.20 | 100.018 |
| 0.25 | 100.021 |
| 0.30 | 100.024 |
| 0.35 | 100.026 |
| 0.40 | 100.029 |
| 0.45 | 100.030 |
| 0.50 | 100.032 |
| 0.55 | 100.033 |
| 0.60 | 100.034 |
| 0.65 | 100.035 |
| 0.70 | 100.036 |
| 0.75 | 100.037 |
| 0.80 | 100.038 |
| 0.85 | 100.038 |
| 0.90 | 100.039 |
| 0.95 | 100.040 |
| 1.00 | 100.040 |

The porous media is subject to the a constant pressure a $x = 0$ of $p = 100$ kPa and an exit flow rate of $Q = 100$ m/s is measured.

Use the code developed in part (a) to help you estimate what the permiablity $\kappa(x)$ this material. You can use a consant viscosity of $\mu = 1$ Pa·s. Clearly indicate your answer and explain your approach (preferably with plots). The total diffusivity coefficient is $K = \frac{\kappa}{\mu}$.

**Note:** Submit a working version of your code to Canvas. Any supplemental material explaining your answer in part (b) can be turned in to me via hard copy or scanned and submitted to Canvas with your code.